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Tightly secure hierarchical identity-based encryption

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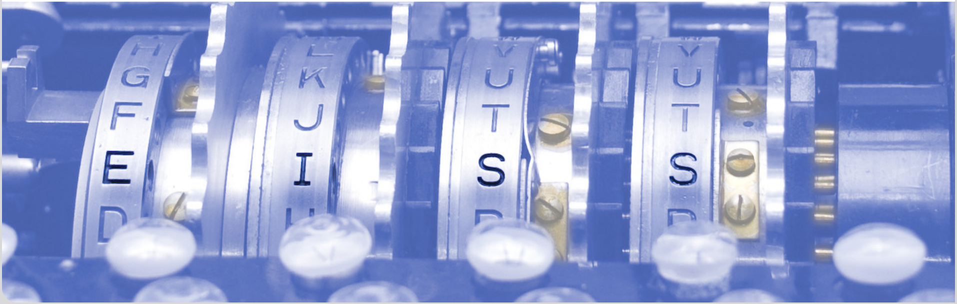


Our main construction

We construct the first HIBE with tight reduction in the standard model.

Scheme 1:

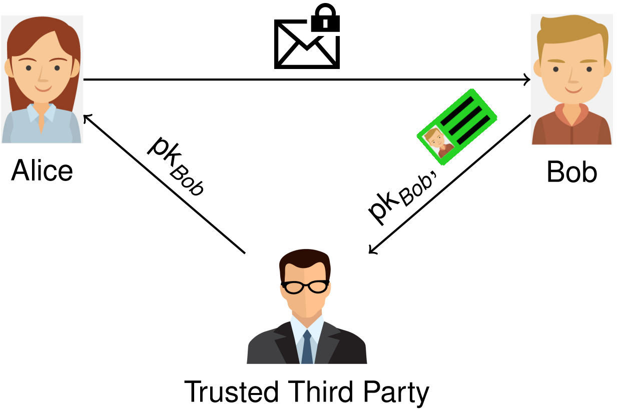
- ✓ $O(1)$ size ciphertexts
- ✗ longer user secret keys

Scheme 2:

- ✗ longer ciphertexts
- ✓ shorter user secret keys

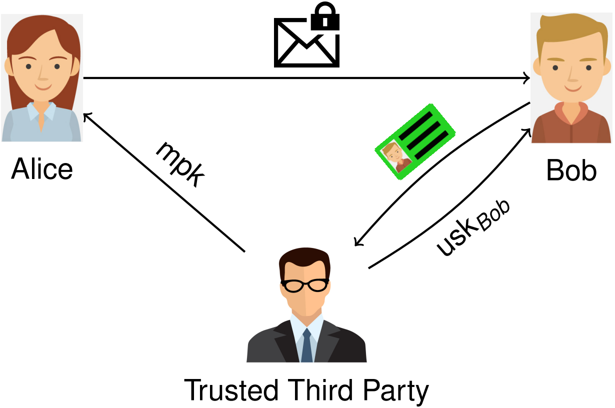
Outline

Public key encryption



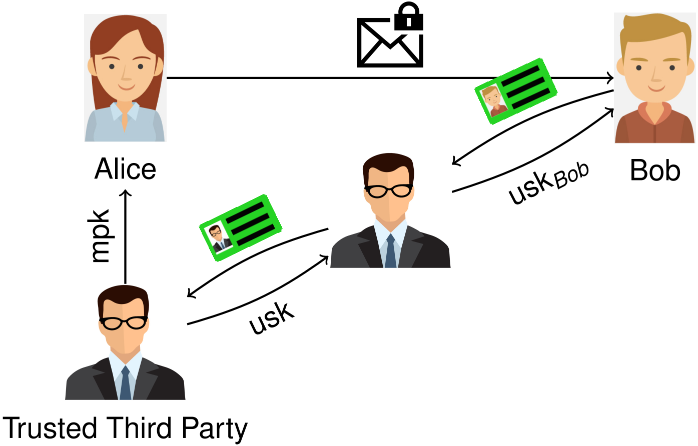
- Alice needs to obtain a public key for each member

Identity-based encryption



- Alice needs to obtain only the master public key
- Encryption with identities (e.g. e-mail address)

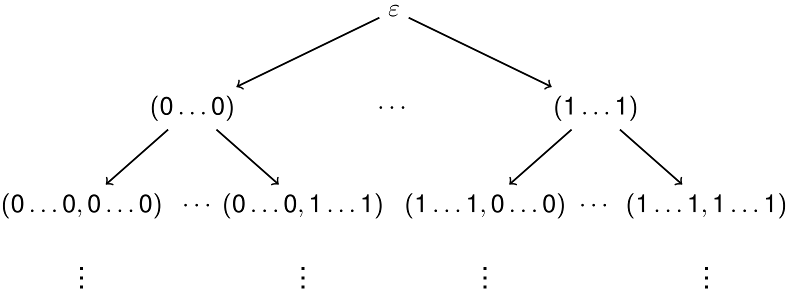
Hierarchical Identity-based encryption



- Hierarchy of key generators

Key delegation

Identities have the form $(\mathrm{id}_1, \ldots, \mathrm{id}_p)$.



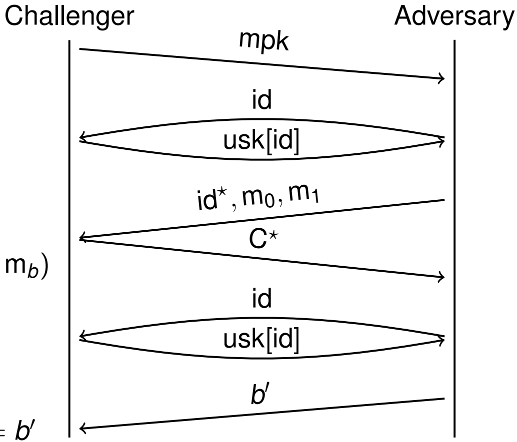
- Each user can generate keys for its children

Security game (IND-HID-CPA)

$b \xleftarrow{\$} \{0,1\}$

$C^\star \xleftarrow{\$} \mathrm{Enc}(\mathrm{mpk}, \mathrm{id}^\star, \mathrm{m}_b)$

$b = b'$



The adversary must not ask user secret keys for prefixes of $\mathrm{id}^\star$.

Tight security

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Scheme (e.g. HIBE)	Reduction →	Static assumption (e.g. Diffie Hellman)

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Can be broken with probability ε using resources ρ.		Can be broken with probability ε/ℓ using resources ρ.

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Security loss ℓ can depend on:

- scheme parameters
  - *L*: maximum hierarchy depth
  - α: bit length of identities
- λ: the security parameter
- the attacker's resources
  - *Q*: # user secret key queries

Larger security loss requires larger security parameter.

Tight security

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- scheme parameters
  - L: maximum hierarchy depth
  - α: bit length of identities
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- the attacker's resources
  - Q: # user secret key queries

Tight security:

- scheme parameters and λ: allowed
- the attacker's resources: not allowed

Larger security loss requires larger security parameter.

History: (H)IBE, tight security

Scheme	Hierarchical	Full Security	No ROM	Tight
[CW13], [BKP14],...	✗	✓	✓	✓
([BKP14])	✓	✗	✓	✓
[BBG05]	✓	✓	✗	(✓)
[Lew12], [BKP14],...	✓	✓	✓	✗
???	✓	✓	✓	✓

Recap of [BKP14]

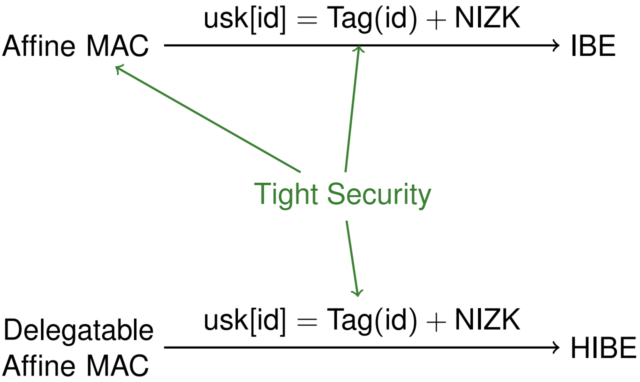
$$\text{Affine MAC} \xrightarrow{\mathsf{usk[id]} = \mathsf{Tag(id)} + \mathsf{NIZK}} \text{IBE}$$

Recap of [BKP14]

Affine MAC $\xrightarrow{\text{usk[id]} = \text{Tag(id)} + \text{NIZK}}$ IBE

Delegatable Affine MAC $\xrightarrow{\text{usk[id]} = \text{Tag(id)} + \text{NIZK}}$ HIBE

Recap of [BKP14]



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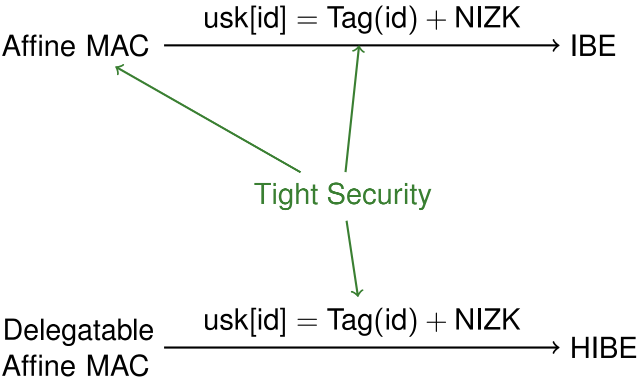

Problem: A Delegatable Affine MAC with tight security

Matrix Decisional Diffie-Hellman assumption

We assume prime order groups $\mathbb{G}_1, \mathbb{G}_2, \mathbb{G}_T$ with pairing $e : \mathbb{G}_1 \times \mathbb{G}_2 \to \mathbb{G}_T$

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Implicit Representation

$$[\mathbf{A}]_s := \begin{pmatrix} a_{1,1}\mathcal{P}_s & \dots & a_{1,m}\mathcal{P}_s \\ & \ddots & \\ a_{n,1}\mathcal{P}_s & \dots & a_{n,m}\mathcal{P}_s \end{pmatrix} \in \mathbb{G}_s^{n \times m},$$

where $s \in \{1, 2, T\}$.

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$\mathcal{D}_k$-MDDH-assumption

$$([\mathbf{A}]_s, [\mathbf{Ar}]_s) \stackrel{c}{\approx} ([\mathbf{A}]_s, [\mathbf{w}]_s)$$

for $\mathbf{A} \stackrel{\$}{\leftarrow} \mathcal{D}_k \subset \mathbb{Z}_q^{n \times k}$ for $k < n$, $\mathbf{r} \stackrel{\$}{\leftarrow} \mathbb{Z}_q^k$ and $\mathbf{w} \stackrel{\$}{\leftarrow} \mathbb{Z}_q^n$

[BKP14]: Affine MACs

- $\mathsf{Gen}_{\mathsf{MAC}}(1^\lambda)$:

$$\mathsf{sk} := \left(\mathbf{B}, \mathbf{x}_1, \ldots, \mathbf{x}_\ell, x_0' \right)$$

matrix distribution

uniform random

[BKP14]: Affine MACs

- Tag($sk_{MAC}, m \in \mathcal{S}$): ($[\mathbf{t}]_2, [u]_2$) with

$$\mathbf{t} = \mathbf{B}\,\mathbf{s}$$

matrix distribution

uniform random

pseudorandom

$$u = \sum_i f_i(m)\,\mathbf{x}_i^\top\,\mathbf{t} + x_0' \qquad (1)$$

Public functions $f_i : \mathcal{M} \to \mathbb{Z}_q$ define different concrete MACs.

[BKP14]: Affine MACs

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Public functions $f_i : \mathcal{M} \to \mathbb{Z}_q$ define different concrete MACs.

- $\mathsf{Ver}_{\mathsf{MAC}}(\mathsf{sk}_{\mathsf{MAC}}, \mathsf{m}, \tau = ([\mathbf{t}]_2, [u]_2))$ checks eq. (1).

[BKP14]: Delegatable Affine MACs

- Public values:

$$\left([\mathbf{B}]_2 , [\mathbf{B}^\top \mathbf{x}_1]_2 , \ldots , [\mathbf{B}^\top \mathbf{x}_\ell]_2 \right)$$

  Necessary for key delegation
- Flexible-length messages $m \in \mathcal{S}^{\leq L}$

Security of Affine MACs

- Security requirement: $[u_{\mathsf{m}}]_2$ is indistinguishable from a random value for m with fixed length.

Security of Affine MACs

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$\mathsf{MAC}_{\mathsf{NR}}[\mathcal{D}_k]$ in [BKP14] achieves this with a tight bit-by-bit randomization:

Proof idea in [BKP14]

- $x'_0 = \mathsf{RF}_0(\varepsilon)$
- $\mathsf{RF}_i(\mathrm{m}_{|i}) \overset{c}{\approx} \mathsf{RF}_{i+1}(\mathrm{m}_{|i+1})$ via

$$\mathsf{RF}_{i+1}(\mathrm{m}_{|i+1}) := \begin{cases} \mathsf{RF}_i(\mathrm{m}_{|i}) & \text{if } \mathrm{m}_{i+1} = 0 \\ \mathsf{RF}_i(\mathrm{m}_{|i}) + \mathsf{RF}'_i(\mathrm{m}_{|i}) & \text{if } \mathrm{m}_{i+1} = 1 \end{cases}$$

Security of Delegatable Affine MACs

- Security requirement: $[u_{\mathsf{m}}]_2$ is indistinguishable from a random value for m with flexible length.

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Security of Delegatable Affine MACs

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The problem with bit-by-bit randomization

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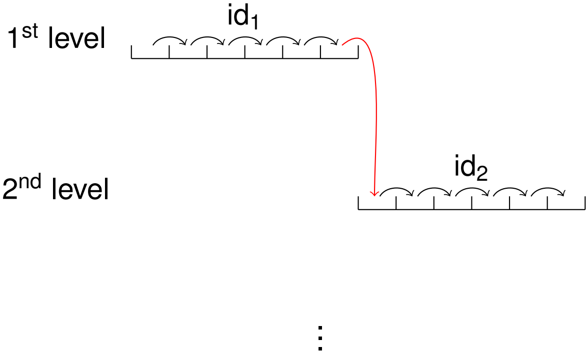
Example

$$\mathsf{RF}_{|\mathsf{id}_1|+1}(\mathsf{id}_1) = \mathsf{RF}_{|\mathsf{id}_1|+1}(\mathsf{id}_1, 0) \quad ↯$$

$\mathsf{MAC}_{\mathsf{NR}}[\mathcal{D}_k]$ in the hierarchical setting



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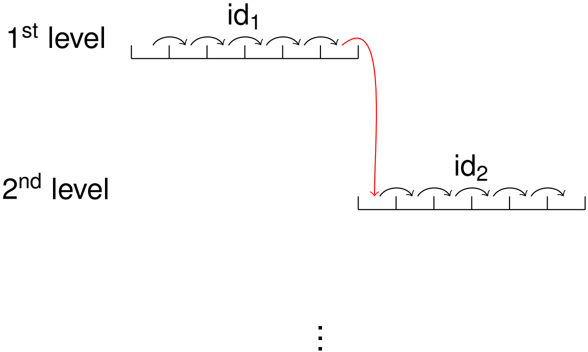

How can we randomize messages with flexible length?

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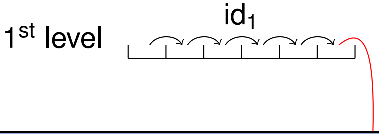

Solution: Independent randomization of each level

$\vdots$

How can we randomize messages with flexible length?

Our new MACs

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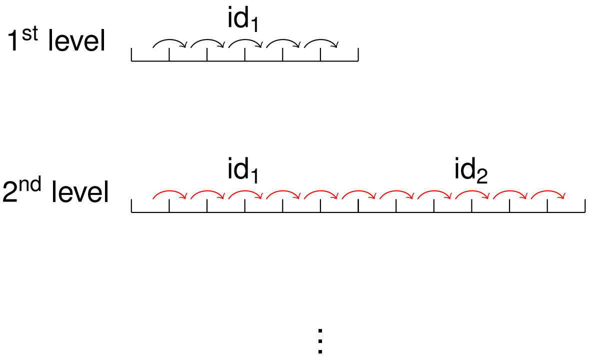
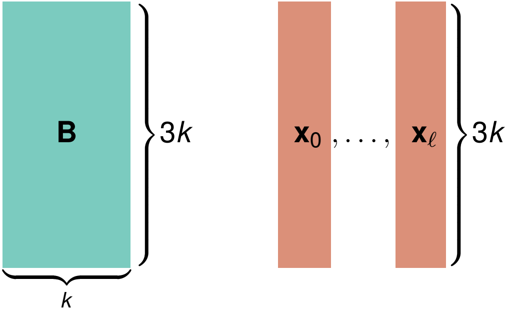


Our new MACs

Randomization technique based on [GHKW16]:

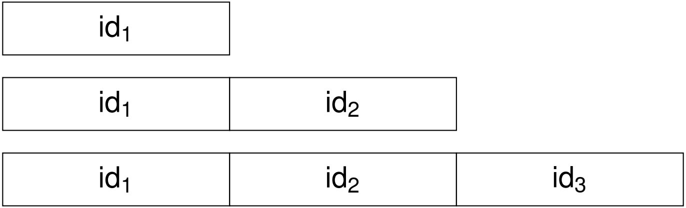


k dimensions are public

$2k$ dimensions are used for randomization

Construction 1

Randomize levels successively



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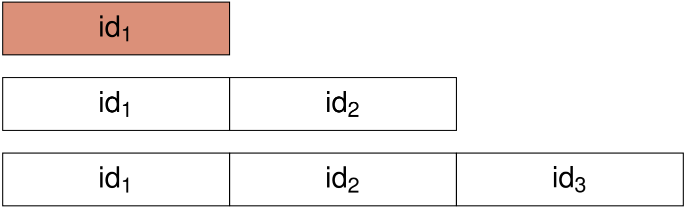

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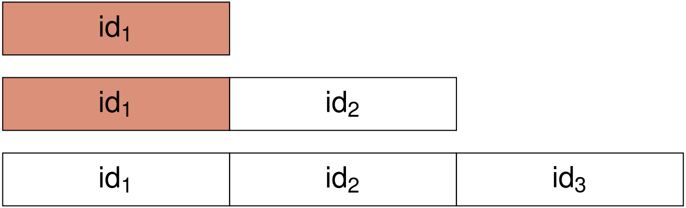

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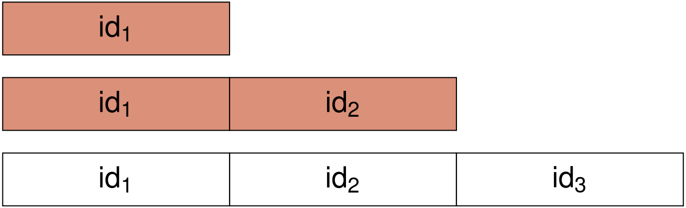

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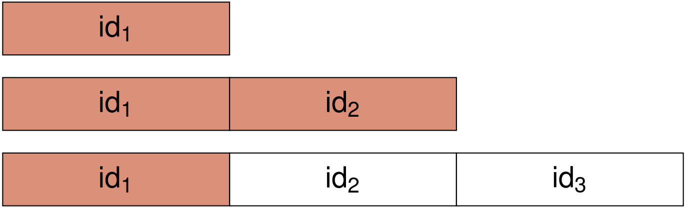

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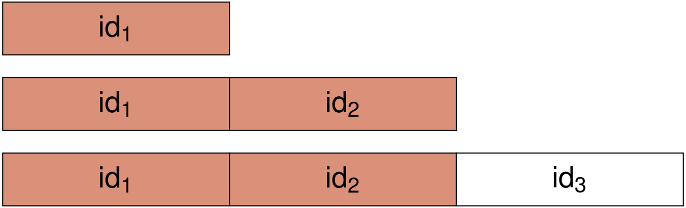

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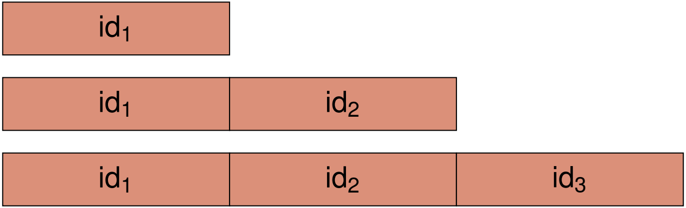
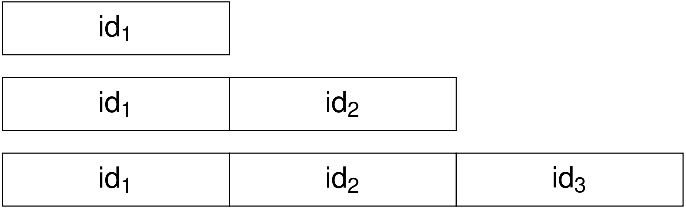


Construction 2

Randomize levels simultaneously



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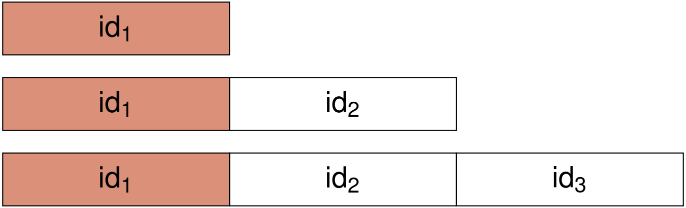

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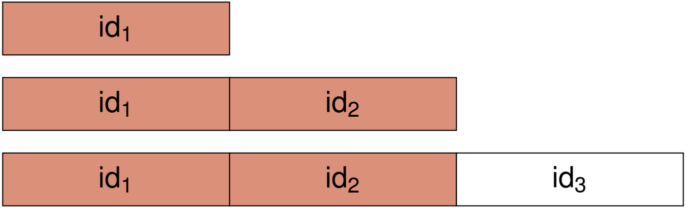

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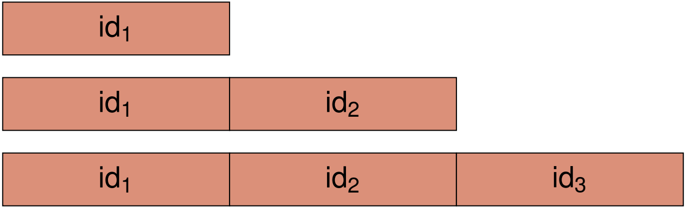

Our new MAC

Construction 1:

- Randomize levels successively
  - $\Rightarrow$ Security loss $O(\alpha L^2)$
- Constant size ciphertexts
- Uses the same **t** on all levels

Construction 2:

- Randomize levels simultaneously
  - $\Rightarrow$ Security loss $O(\alpha L)$
- $O(L)$ size ciphertexts
- Requires different values for **t** on each level
- Requires a generalization of the [BKP14] transformation

Overview of HIBE schemes

with full security and without random oracles.

Scheme	$\lvert mpk\rvert$	$\lvert usk\rvert$	$\lvert C\rvert$	Loss	Assumption
[Wat05]	$O(\alpha L)$	$O(\alpha L)$	$O(L)$	$O(\alpha Q)^L$	DBDH
[Wat09]	$O(L)$	$O(L)$	$O(L)$	$O(Q)$	2-LIN
[Lew12]	$O(1)$	$O(L)$	$O(L)$	$O(Q)$	2-LIN
[CW13]	$O(Lk^2)$	$O(Lk)$	$O(k)$	$O(Q)$	k-LIN
[BKP14]	$O(Lk^2)$	$O(Lk)$	$O(k)$	$O(Q)$	k-LIN
[GCTC16]	$O(1)$	$O(L)$	$O(L)$	$O(Q)$	SXDH
Ours (v. 1)	$O(\alpha L^2)$	$O(\alpha L^2)$	$O(1)$	$O(\alpha L^2)$	SXDH
Ours (v. 2)	$O(\alpha L^2)$	$O(L)$	$O(L)$	$O(\alpha L)$	SXDH

$\lvert mpk\rvert$, $\lvert usk\rvert$, $\lvert C\rvert$ are in number of group elements

- L: maximum hierarchy depth
- α: bit length of identities
- Q: # user secret key queries

Summary

- First tightly secure HIBE schemes in standard model
  - based on MDDH (e.g. SXDH or k-LIN) assumption

Core idea

New randomization technique for flexible length identities

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Implications

- tightly CCA-secure (H)IBE (via the CHK transformation using one-time signatures) and

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Core idea

New randomization technique for flexible length identities

Implications

- tightly CCA-secure (H)IBE (via the CHK transformation using one-time signatures) and
- the first tightly secure (hierarchical) identity-based signature scheme (via the Naor transformation).

Open Problems

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- Tight security in multi instance, multi challenge setting?
- Shorter public parameters?

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Pictures

Alice, Bob, Trusted Party: freepik.com
Encrypted Mail: Icon made by SimpleIcon from www.flaticon.com